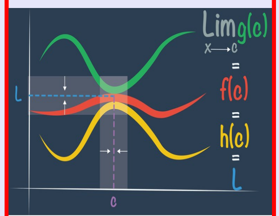
Calculus I

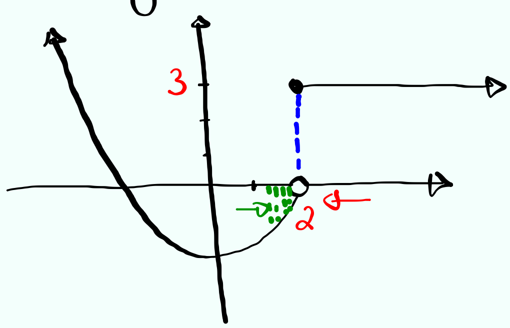
Lecture 2



Feb 19-8:47 AM

Class QZ 2

Use the graph below for $y = f(x)$



1) $\lim\limits_{x \to 2^+} f(x) = 3$

2) $\lim\limits_{x \to 2^-} f(x) = 0$

3) $\lim\limits_{x \to 2} f(x) =$ D.N.E.

4) $f(2) = 3$

Jan 5-12:08 PM

Evaluate

$\frac{\pi}{4} = 45^\circ$

$\tan 45^\circ = 1$

1) $\lim_{x \to 1} \frac{x-1}{x+1} = \frac{1-1}{1+1} = \frac{0}{2} = \boxed{0}$

2) $\lim_{x \to 0} \left(2 - 2\tan\left(x + \frac{\pi}{4}\right)\right) = 2 - 2\tan\left(0 + \frac{\pi}{4}\right)$

$= 2 - 2\tan\frac{\pi}{4} = 2 - 2\cdot 1 = \boxed{0}$

3) $\lim_{x \to 5} \frac{x^3 - 125}{x^2 - 25} = \frac{5^3 - 125}{5^2 - 25} = \frac{0}{0}$ I.F.

π Rad $= 180^\circ$

$\frac{\pi}{2}$ Rad. $= 90^\circ$

$\frac{\pi}{4}$ Rad $= 45^\circ$

$= \lim_{x \to 5} \frac{\cancel{(x-5)}(x^2 + 5x + 25)}{\cancel{(x-5)}(x+5)}$

$= \lim_{x \to 5} \frac{x^2 + 5x + 25}{x+5} = \frac{5^2 + 5(5) + 25}{5+5} = \frac{75}{10} = \boxed{7.5} = \boxed{\frac{15}{2}}$

Jan 6-8:05 AM

4) $\lim_{x \to 25} \frac{x - 25}{\sqrt{x} - 5} = \frac{25 - 25}{\sqrt{25} - 5} = \frac{0}{5-5} = \frac{0}{0}$ I.F.

$= \lim_{x \to 25} \frac{(x-25)(\sqrt{x}+5)}{\underbrace{(\sqrt{x}-5)}_{A-B}\underbrace{(\sqrt{x}+5)}_{A+B}} = \lim_{x \to 25} \frac{\cancel{(x-25)}(\sqrt{x}+5)}{\underbrace{\cancel{x-25}}_{A^2-B^2}}$

$= \lim_{x \to 25} (\sqrt{x} + 5) = \sqrt{25} + 5$

$= \boxed{10}$

5) $\lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3} = \frac{\frac{1}{3} - \frac{1}{3}}{3 - 3} = \frac{0}{0}$ I.F.

Use LCD $= 3x$

$= \lim_{x \to 3} \frac{3x\left(\frac{1}{x} - \frac{1}{3}\right)}{3x(x-3)} = \lim_{x \to 3} \frac{3\cancel{x}\cdot\frac{1}{\cancel{x}} - \cancel{3}x\cdot\frac{1}{\cancel{3}}}{3x(x-3)} = \lim_{x \to 3} \frac{\overset{-1}{\cancel{3-x}}}{3x\cancel{(x-3)}}$

$= \lim_{x \to 3} \frac{-1}{3x} = \boxed{\frac{-1}{9}}$

Jan 6-8:16 AM

Evaluate $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ for any quadratic function.

Recall $(A+B)^2 = A^2 + 2AB + B^2$

$f(x) = ax^2 + bx + c, \; a \neq 0$

$$\lim_{h \to 0} \frac{a(x+h)^2 + b(x+h) + c - (ax^2+bx+c)}{h}$$

$$= \lim_{h \to 0} \frac{a(x^2+2xh+h^2) + b(x+h) + c - ax^2 - bx - c}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{ax^2} + 2axh + ah^2 + \cancel{bx} + bh \cancel{+c} \cancel{-ax^2} \cancel{-bx} \cancel{-c}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2ax + ah + b)}{\cancel{h}} = \lim_{h \to 0} (2ax + \cancel{ah}^{\,0} + b)$$

$$= \boxed{2ax + b}$$

Jan 6-8:27 AM

Evaluate $\lim\limits_{x \to h} \dfrac{f(x) - f(h)}{x - h}$ for any linear function.

$f(x) = mx + b$

$$= \lim_{x \to h} \frac{mx + b - (mh + b)}{x - h}$$

$$= \lim_{x \to h} \frac{mx \cancel{+b} - mh \cancel{-b}}{x - h} = \lim_{x \to h} \frac{m\cancel{(x-h)}}{\cancel{x-h}}$$

$$= \boxed{m}$$

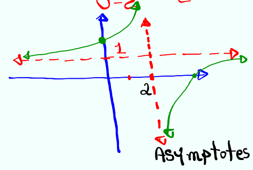
Jan 6-8:35 AM

Given $f(x) = \frac{x-4}{x-2}$

1) Domain $x-2 \neq 0$ All Reals except 2.
$x \neq 2$ $(-\infty, 2) \cup (2, \infty)$

2) x-Int $\to y=0 \to f(x)=0$ $\frac{x-4}{x-2}=0$ $x-4=0$
$(4,0)$ $\boxed{x=4}$

3) y-Int. $\to x=0 \to y=f(0)=\frac{0-4}{0-2}=\frac{-4}{-2}=\boxed{2}$
$(0,2)$



Asymptotes

V.A. $x=2$

H.A. $y=1$

4) $\lim\limits_{x \to 2^+} f(x) = \boxed{-\infty}$

$\lim\limits_{x \to 2^-} f(x) = \boxed{\infty}$

$\lim\limits_{x \to \infty} f(x) = \boxed{1}$

$\lim\limits_{x \to -\infty} f(x) = \boxed{1}$

For $x=1000$

$f(1000) = \frac{1000-4}{1000-2}$

$\approx .997$

For $x = 10000$

$f(10000) = \frac{10000-4}{10000-2}$

$\approx .9997$

As $x \to \infty$, $f(x) \to 1$

Jan 6-8:42 AM

Evaluate $\lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$ for $f(x) = x^3$

$= \lim\limits_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim\limits_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$

$= \lim\limits_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}} = \lim\limits_{h \to 0} (3x^2 + \cancelto{0}{3xh} + \cancelto{0}{h^2})$

$= \boxed{3x^2}$

$(1+2)^3 = 1^3 + 2^3 = 1 + 8 = 9$

$3^3 = 27$

$(A+B)^3 = (A+B)(A+B)(A+B)$

$= (A+B)(A^2 + 2AB + B^2)$

$= A^3 + 2A^2B + AB^2 + A^2B + 2AB^2 + B^3$

$= \boxed{A^3 + 3A^2B + 3AB^2 + B^3}$

$(A+B)^4 = A^4 + 4A^3B + 6A^2B^2 + 4AB^3 + B^4$

Jan 6-8:55 AM

Evaluate $\lim\limits_{x\to 2} \dfrac{f(x)-f(2)}{x-2}$ for $f(x)=x^3+3x^2+8x+20$.

$f(2)=2^3+3(2)^2+8(2)+20$
$=56$

$$\lim_{x\to 2}\frac{x^3+3x^2+8x+20-56}{x-2}$$

$$=\lim_{x\to 2}\frac{x^3+3x^2+8x-36}{x-2}=\frac{2^3+3(2)^2+8(2)-36}{2-2}=\frac{0}{0}$$

I.F.

$$=\lim_{x\to 2}\frac{\cancel{(x-2)}(x^2+5x+18)}{\cancel{x-2}}=\lim_{x\to 2}(x^2+5x+18)$$

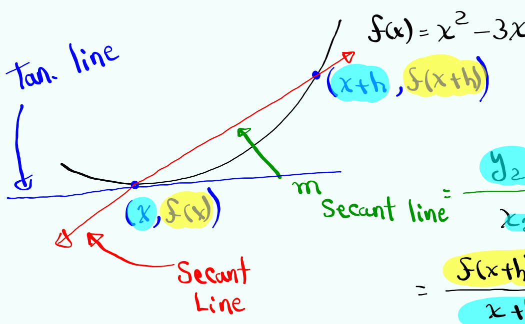
$=2^2+5(2)+18$

$=\boxed{32}$

Synthetic Division

2	1	3	8	-36
		2	10	36
	1	5	18	0

Jan 6-9:10 AM



$$m_{\text{Secant line}}=\frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{f(x+h)-f(x)}{x+h-x}$$

$$=\frac{f(x+h)-f(x)}{h}$$

For our Problem

$$\frac{(x+h)^2-3(x+h)-(x^2-3x)}{h}=\frac{\cancel{x^2}+2xh+h^2\cancel{-3x}-3h\cancel{-x^2}\cancel{+3x}}{h}$$

$$=\frac{\cancel{h}(2x+h-3)}{\cancel{h}}=\boxed{2x+h-3}$$

$$m_{\text{tan. line}}=\lim_{h\to 0} m_{\text{Secant line}}$$

$$m_{\text{tan. line}}=\lim_{h\to 0}(2x+\overset{\to 0}{\cancel{h}}-3)=\boxed{2x-3}$$

Jan 6-9:22 AM

Class QZ 3

Box Your Final Ans.

Solve $3x^2 - 5x = 2$ using the quadratic Formula.

Quadratic Eqn

$ax^2 + bx + c = 0, \; a \neq 0$

$3x^2 - 5x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-2)}}{2(3)} = \frac{5 \pm \sqrt{25 + 24}}{6} = \frac{5 \pm \sqrt{49}}{6} = \frac{5 \pm 7}{6}$$

$$x = \frac{5+7}{6} = \frac{12}{6} = \boxed{2} \qquad x = \frac{5-7}{6} = \frac{-2}{6} = \boxed{-\frac{1}{3}}$$

Jan 6-9:32 AM

Rules of limits:

1) $\lim_{x \to a} c = c$

2) $\lim_{x \to a} x = a$

3) $\lim_{x \to a} x^n = a^n$

4) $\lim_{x \to a} cf(x) = c \lim_{x \to a} f(x)$

5) $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

6) $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

Jan 6-10:07 AM

Find $\lim\limits_{x \to 2} 10 = 10$ constant

Find $\lim\limits_{x \to -2} x^4 = (-2)^4 = \boxed{16}$

Find $\lim\limits_{x \to 100} \frac{1}{10}\sqrt{x} = \frac{1}{10} \lim\limits_{x \to 100} \sqrt{x} = \frac{1}{10} \cdot \sqrt{100} = \frac{1}{10} \cdot 10 = \boxed{1}$

Suppose $\lim\limits_{x \to 4} f(x) = 10$ & $\lim\limits_{x \to 4} g(x) = -3$, find

1) $\lim\limits_{x \to 4} -5f(x)$

$= -5 \cdot \lim\limits_{x \to 4} f(x)$

$= -5 \cdot 10$

$= \boxed{-50}$

2) $\lim\limits_{x \to 4} [f(x) - g(x)]$

$= \lim\limits_{x \to 4} f(x) - \lim\limits_{x \to 4} g(x)$

$= 10 - (-3) = \boxed{13}$

Jan 6-10:11 AM

Evaluate $\lim\limits_{x \to -5} (x^2 - 10x + 25)$

$= \lim\limits_{x \to -5} x^2 - \lim\limits_{x \to -5} 10x + \lim\limits_{x \to -5} 25$

$= (-5)^2 - 10 \lim\limits_{x \to -5} x + 25$

$= 25 - 10(-5) + 25 = \boxed{100}$

Jan 6-10:18 AM

Find $\lim_{x\to 8} f(x)$ & $\lim_{x\to 8} g(x)$ if

$\lim_{x\to 8} [f(x) + g(x)] = 7$

$\lim_{x\to 8} [f(x) - g(x)] = 3$

$\Rightarrow \begin{cases} \lim_{x\to 8} f(x) + \cancel{\lim_{x\to 8} g(x)} = 7 \\ \lim_{x\to 8} f(x) - \cancel{\lim_{x\to 8} g(x)} = 3 \end{cases}$

$2 \lim_{x\to 8} f(x) = 10$

$\boxed{\lim_{x\to 8} f(x) = 5}$

Solve

$\begin{cases} A + B = 7 \\ A - B = 3 \end{cases}$

$2A = 10$

$\boxed{A = 5}$

$5 + B = 7$

$\boxed{B = 2}$

$\lim_{x\to 8} f(x) + \lim_{x\to 8} g(x) = 7$

$5 + \lim_{x\to 8} g(x) = 7$

$\boxed{\lim_{x\to 8} g(x) = 2}$

Jan 6-10:20 AM

More rules:

7) $\lim_{x\to a} [f(x) \cdot g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$

8) $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, $\lim_{x\to a} g(x) \neq 0$

Evaluate $\lim_{x\to 0} [(x^2 + 5) \cos x]$

$= \lim_{x\to 0} (x^2+5) \cdot \lim_{x\to 0} \cos x$

$= (0^2 + 5) \cdot \boxed{\cos 0} \rightarrow 1$

$= 5 \cdot 1 = \boxed{5}$

Jan 6-10:27 AM

Evaluate $\lim_{x \to \pi} \frac{\cos x - 1}{1 + \sin x}$

$\pi \text{ Rad} = 180°$

$$= \frac{\lim_{x \to \pi} [\cos x - 1]}{\lim_{x \to \pi} [1 + \sin x]} = \frac{\cos \pi - 1}{1 + \sin \pi}$$

$\cos \pi \to -1$, $\sin \pi \to 0$

$$= \frac{-1 - 1}{1 + 0} = \frac{-2}{1} = \boxed{-2}$$

Jan 6-10:32 AM

Evaluate $\lim_{x \to -3} \frac{x^2 f(x) - 4}{5x + f(x)}$ if $\lim_{x \to -3} f(x) = -5$.

$$= \frac{\lim_{x \to -3} [x^2 f(x) - 4]}{\lim_{x \to -3} [5x + f(x)]} = \frac{\lim_{x \to -3} x^2 f(x) - \lim_{x \to -3} 4}{\lim_{x \to -3} 5x + \lim_{x \to -3} f(x)}$$

$$= \frac{(-3)^2 \cdot (-5) - 4}{5(-3) - 5} = \frac{9(-5) - 4}{-15 - 5}$$

$$= \frac{-49}{-20} = \boxed{\frac{49}{20}}$$

Jan 6-10:35 AM

Given

$$f(x)=\begin{cases}-x+1 & \text{if } x<1\\ \sqrt{x-1} & \text{if } x\geq 1\end{cases}$$

Find

$\lim\limits_{x\to -2} f(x) = -(-2)+1 = \boxed{3}$

$-2<1$

$\lim\limits_{x\to 2} f(x) = \sqrt{2-1} = \sqrt{1} = \boxed{1}$

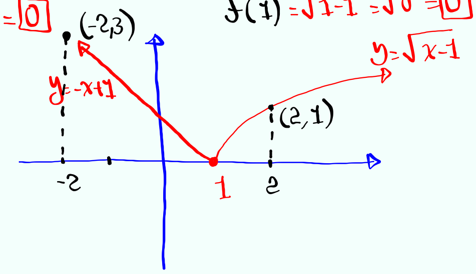
$2\geq 1$

$\lim\limits_{x\to 1^-} f(x) = -1+1 = \boxed{0}$

$\lim\limits_{x\to 1^+} f(x) = \sqrt{1-1} = \sqrt{0} = \boxed{0}$

$\lim\limits_{x\to 1} f(x) = \boxed{0}$

$f(1) = \sqrt{1-1} = \sqrt{0} = \boxed{0}$



Jan 6-10:43 AM

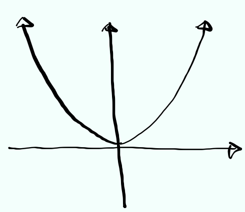
Continuity

$f(x)$ is continuous at $x=a$ when

$$\lim_{x\to a} f(x) = f(a)$$

Discontinuous happen when there is a gap, a hole, or a jump.

Is $f(x)=x^2$ cont. at $x=2$?



No Jump, No gap, No hole

Yes

$\lim\limits_{x\to 2} f(x) = \lim\limits_{x\to 2} x^2 = 2^2 = 4$

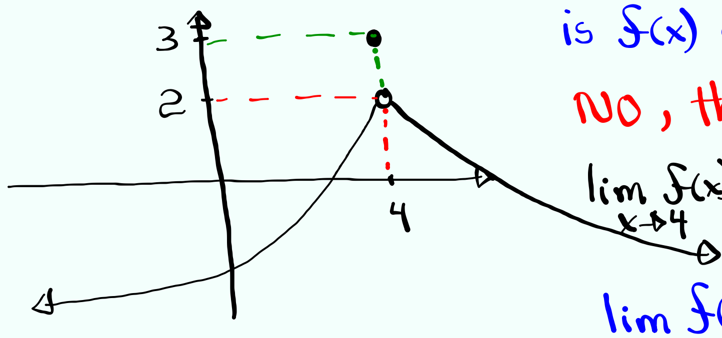
$f(2) = 2^2 = 4$

Since $\lim\limits_{x\to 2} f(x) = f(2)$

then it is Cont. at $x=2$.

Jan 6-10:54 AM

Consider the graph of $y=f(x)$ below



is $f(x)$ cont. at $x=4$?

NO, there is a hole.

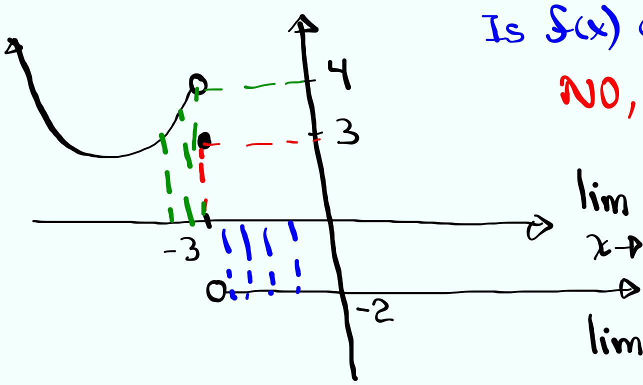
$\lim\limits_{x\to4} f(x) = 2$, $f(4)=3$

$\lim\limits_{x\to4} f(x) \neq f(4)$

Discontinuous at $x=4$

Jan 6-10:59 AM

Consider the graph of $y=f(x)$ below



Is $f(x)$ cont. at $x=-3$?

NO, gap, Jump, hole

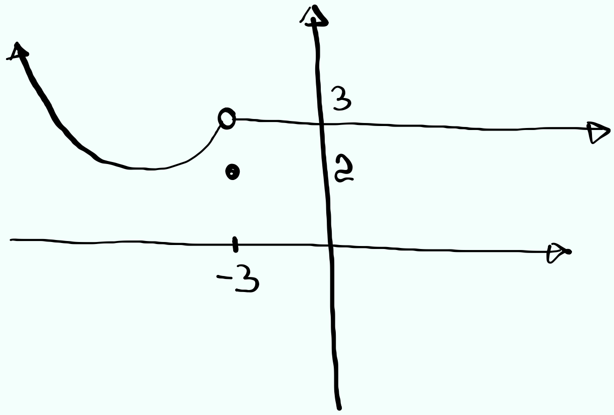
$\lim\limits_{x\to-3^-} f(x) = 4$

$\lim\limits_{x\to-3^+} f(x) = -2$

$\lim\limits_{x\to-3} f(x) =$ D.N.E.

$f(-3) = 3$

Jan 6-11:02 AM



$\lim_{x \to -3^-} f(x) = 3$

$\lim_{x \to -3^+} f(x) = 3$

$\lim_{x \to -3} f(x) = 3$

$f(-3) = 2$

Since $\lim_{x \to -3} f(x) \neq f(-3)$

It is discontinuous at $x=-3$.

Jan 6-11:06 AM

Suppose $\sqrt{x}+1 \leq f(x) \leq x^2+1$ near $x=1$.

Find $\lim_{x \to 1} f(x)$.

$\lim_{x \to 1} (\sqrt{x}+1) = \sqrt{1}+1 = \boxed{2}$

$\lim_{x \to 1} f(x) = 2$

$\lim_{x \to 1} (x^2+1) = 1^2+1 = \boxed{2}$

Squeeze Theorem

If $h(x) \leq f(x) \leq g(x)$ near $x=a$ and

$\lim_{x \to a} h(x) = \lim_{x \to a} g(x) = L$, then $\lim_{x \to a} f(x) = L$.

Jan 6-11:10 AM

Suppose $1+\cos x \le f(x) \le x^2+2$ near $x=0$,

+2

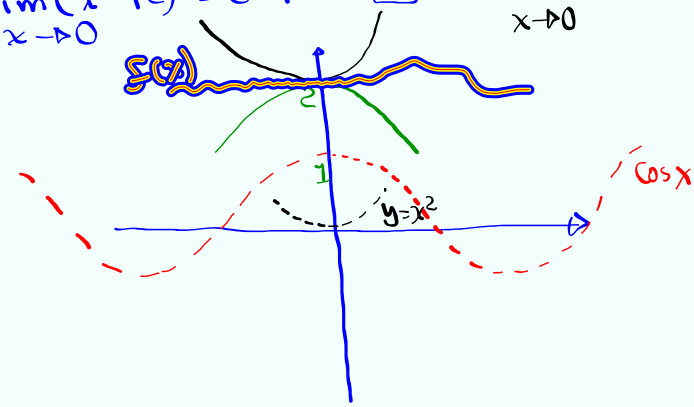
find $\lim_{x\to 0} f(x)$.

$\lim_{x\to 0}(1+\cos x) = 1+\cos 0 = 1+1 = \boxed{2}$

$\lim_{x\to 0}(x^2+2) = 0^2+2 = \boxed{2}$

By S.T.,

$\lim_{x\to 0} f(x) = \boxed{2}$



Jan 6-11:16 AM

Given $-x^2 < f(x) < |x|$

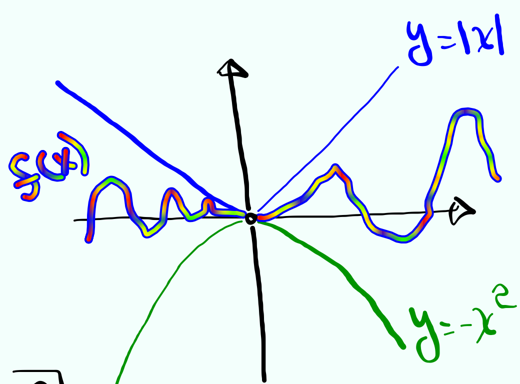
Find $\lim_{x\to 0} f(x)$.

$\lim_{x\to 0} -x^2 = 0$

$\lim_{x\to 0} |x| = 0$

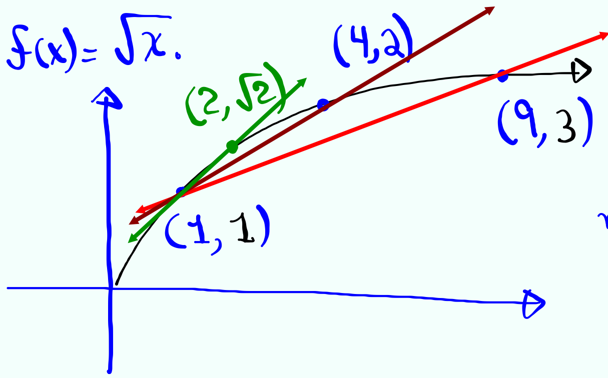
By S.T.,

$\lim_{x\to 0} f(x) = \boxed{0}$



Jan 6-11:23 AM

Find the slope of the secant line for $x=1$ & $x=9$ with $f(x)=\sqrt{x}$.



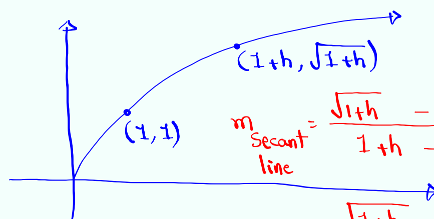
$$m=\frac{3-1}{9-1}=\frac{2}{8}=\frac{1}{4}=\boxed{.25}$$

$$m=\frac{2-1}{4-1}=\boxed{\frac{1}{3}=.\overline{3}}$$

what about $x=1$ & $x=4$.

what about $x=1$ & $x=2$. $m=\dfrac{\sqrt{2}-1}{2-1}=\boxed{.414}$

Jan 6-11:28 AM



$$m_{\text{secant line}}=\frac{\sqrt{1+h}-1}{1+h-1}=\frac{\sqrt{1+h}-1}{h}$$

when $h=0$, secant line becomes tan. line $\dfrac{\sqrt{1+0}-1}{} = \dfrac{0}{0}$ I.F.

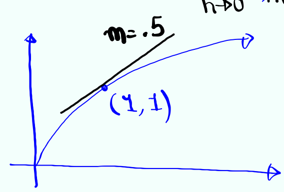
$$\lim_{h\to 0}\frac{\sqrt{1+h}-1}{h}=\lim_{h\to 0}\frac{(\overset{A}{\sqrt{1+h}}\overset{-B}{-1})(\overset{A}{\sqrt{1+h}}\overset{+B}{+1})}{h(\sqrt{1+h}+1)}$$

$$=\lim_{h\to 0}\frac{\overset{A^2}{(\sqrt{1+h})^2}-\overset{B^2}{(1)^2}}{h(\sqrt{1+h}+1)}$$

$$=\lim_{h\to 0}\frac{\cancel{1}+\cancel{h}^{\,1}\cancel{-1}}{\cancel{h}(\sqrt{1+h}+1)}=\lim_{h\to 0}\frac{1}{\sqrt{1+h}+1}$$

$$=\frac{1}{\sqrt{1+0}+1}=\frac{1}{\sqrt{1}+1}=\frac{1}{2}=\boxed{.5}$$

slope of the tan. line at $x=1$



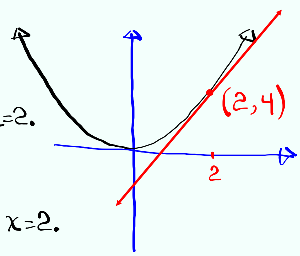
Jan 6-11:35 AM

Given $f(x) = x^2$

1) Graph $f(x)$ ✓

2) label the point with $x=2$.

3) Draw the tan. line at $x=2$.



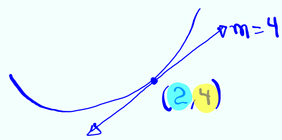
4) Find slope of that tan. line.

$$m_{\text{tan. line}} = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$$

$$= \lim_{h\to 0} \frac{(2+h)^2 - 4}{h} = \lim_{h\to 0} \frac{\cancel{4}+4h+h^2 \cancel{-4}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4+h)}{\cancel{h}}$$

$$= \lim_{h\to 0}(4+h) = \boxed{4}$$



Eqn of tan. line

$y - y_1 = m(x - x_1)$

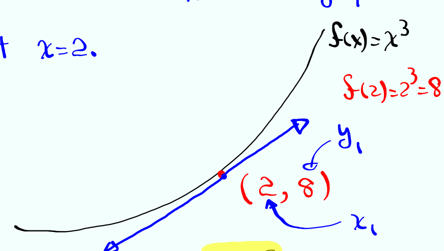
Point-Slope Formula

$y - 4 = 4(x - 2)$

$\boxed{y = 4x - 4}$

Jan 6-11:45 AM

Find eqn of the tan. line to the graph of $f(x) = x^3$ at $x=2$.



$$m = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{(2+h)^3 - 8}{h}$$

$$= \lim_{h\to 0} \frac{2^3 + 3\cdot 2^2 h + 3\cdot 2\cdot h^2 + h^3 - 8}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{8} + 12h + 6h^2 + h^3 \cancel{-8}}{h} = \lim_{h\to 0} \frac{\cancel{h}(12+6h+h^2)}{\cancel{h}}$$

$$= \lim_{h\to 0}(12 + \cancel{6h}^{\to 0} + \cancel{h^2}^{\to 0}) = \boxed{12}$$

Use Point-Slope formula

$y - y_1 = m(x - x_1)$

$y - 8 = 12(x - 2)$

$\boxed{y = 12x - 16}$

Jan 6-12:00 PM

$(A+B)^3 = 1A^3 + 3A^2B + 3AB^2 + 1B^3$

$(2+h)^3 = 2^3 + 3\cdot 2^2 \cdot h + 3\cdot 2 \cdot h^2 + h^3$

$(A+B)^5 = 1A^5 + 5A^4B + 10A^3B^2 + 10A^2B^3 + 5AB^4 + 1B^5$

Simplify $\dfrac{x^5 - 4x^4 + 10x^3 - 8x + 1}{x-1}$

1) Do long division $= \boxed{x^4 - 3x^3 + 7x^2 + 7x - 1}$

2) Synthetic division

$$\begin{array}{c|cccccc} 1 & 1 & -4 & 10 & 0 & -8 & 1 \\ & & 1 & -3 & 7 & 7 & -1 \\ \hline & 1 & -3 & 7 & 7 & -1 & 0 \end{array}$$

Jan 6-12:08 PM